The Form of the Cosmic Gamma-Ray Production Spectrum From Galactic Cosmic-Ray Interactions for Gamma-Ray Energies Greater Than 1 GeV: The Effect of the Decay of Nucleon Isobars and Hyperons on the Cosmic Gamma-Ray Spectrum

6–1 INTRODUCTION

Several workers (Pollack and Fazio, 1963; Ginzburg and Syrovatskii, 1964; Gould and Burbidge, 1965; Hayakawa et al., 1964; Garmire and Kraushaar, 1965; Lieber, Milford, and Spergel, 1965; Dilworth, Maraschi, and Perola, 1968; and Stecker, Tsuruta, and Fazio, 1968) have discussed the production of cosmic γ-rays by the collision of high-energy protons with interstellar and intergalactic gas. Many others have studied cosmic-ray pion production in high-energy interactions. Out of these studies have come various phenomenological models for pion production, sometimes referred to as "pionization." Among these models are the hydrodynamical and "fireball" models. (For a general review, see Fujimoto and Hayakawa, 1967; see also the discussion in ch. 5.) Gamma rays result through the reactions

$$p + p \rightarrow p + p + \text{pions} \tag{6–1}$$

followed by

$$\pi^0 \rightarrow \gamma + \gamma$$

Pal and Peters (Peters, 1962, and Pal and Peters, 1964) have pointed out the significance of the production and decay of hyperons and excited baryons (isobars) as a source of pions in high-energy interactions, which may be of particular importance in producing high-energy pions. In chapter 1 we discussed the details of the γ-ray spectra generated from the formation of various secondary particles produced in p–p interactions. These calculations and the results of recent proton accelerator experiments indicate that in high-energy p–p interactions, the production of the hyperons Σ^+, Σ^0, and Λ and unstable baryon resonances $N^*(1.410)$ and $N^*(1.688)$ and their subsequent decay may be important sources of

very-high-energy γ-ray photons. Although the production cross sections for these particles are relatively low, a substantial fraction of the initial interaction energy is transferred to these particles and hence to the resultant γ-radiation. In this chapter we will use the new accelerator data to calculate the cosmic γ-ray energy spectrum caused by the decay of these particles and to determine at what γ-ray energies the decay of these particles may provide a significant source of cosmic γ-rays. Our discussion will be based on that given by Stecker et al. (1968).

6-2 GAMMA-RAY PRODUCTION MECHANISMS

The basic types of production reactions that we will consider in addition to the usual process denoted by expressions (6–1) are hyperon production:

$$p + p \rightarrow N + Y + K + \text{bosons} \tag{6–2}$$

and nucleon isobar production:

$$p + p \rightarrow N^{*+} + p + \text{bosons} \tag{6–3}$$

The symbols N, Y, K, and N^{*+} denote a nucleon (proton, neutron), a hyperon, a kaon, and a positive nucleon isobar, respectively. Experimental data are available for these reactions for incident-proton energies up to 30 GeV. These reactions produce γ-rays primarily by the decay schemes

$$\begin{aligned} \Sigma^0 &\rightarrow \Lambda + \gamma \\ \Lambda &\rightarrow n + \pi^0 \\ &\qquad \hookrightarrow 2\gamma \\ \Sigma^+ &\rightarrow p + \pi^0 \\ &\qquad \hookrightarrow 2\gamma \end{aligned} \tag{6–4}$$

for the hyperons, and

$$\begin{aligned} N^{*+} &\rightarrow p + \pi^0 \\ &\qquad \hookrightarrow 2\gamma \end{aligned} \tag{6–5}$$

for the isobars. A particular nucleon isobar of isotopic spin $\frac{1}{2}$ will be denoted by the symbol $N^*(M^*)$, where the symbol M^* will be taken to denote its most probable rest mass in GeV.

These decay schemes are of two basic types: The neutral sigma decay, which directly produces one γ-ray, and decays that we will designate as being of type i which indirectly produce two γ-rays from

the decay of a heavy baryon, here denoted by $B_i (i = \Sigma^+, \Lambda, N^{*+})$, as follows:

$$\begin{array}{l} B_i \rightarrow N + \pi^0 \\ \qquad\quad \hookrightarrow 2\gamma \end{array} \tag{6-6}$$

This decay scheme will be referred to, as before, as an "i-process."

6-3 THE DIFFERENTIAL PRODUCTION CROSS SECTIONS OF HYPERONS AND NUCLEON ISOBARS

In order to calculate the cosmic γ-ray spectrum produced from the decay of hyperons and nucleon isobars, we must first determine the differential production cross sections $\sigma_s(E_s|E_p)$, which are defined as the cross sections for the production of a secondary particle of type s with energy E_s by a proton of energy E_p colliding with a proton at rest. These differential production cross sections are determined from proton accelerator experiments and cosmic-ray shower studies. Such studies have indicated that in high-energy collisions the forward baryon retains most of its energy, between 50 and 65 percent. (See, e.g., Baradzei et al., 1962, and Daniel et al., 1963.) This baryon is strongly directed forward, a tendency manifested even at accelerator energies (Morrison, 1963). We will therefore approximate the differential production cross sections for the forwardly produced baryons by the expression

$$\sigma_s(E_s|E_p) = \tfrac{1}{2}\,\sigma_s(E_p)\,\delta[E_s - \eta_s(E_p)] \tag{6-7}$$

where $\eta_s = \chi(E_p - M_p) + M_s$.

Thus, χ designates the fraction of initial energy carried off by the forward baryon, which has been excited to a nucleon isobar N^* or hyperon Y. After deexcitation it is thus equal to the fraction of incident energy carried off by the forward nucleon *plus* the most energetic pion. We will take χ to be a constant and equal to 0.6 in all cases.[1]

The factor of $\frac{1}{2}$ in equation (6–7) is included to take account of the fact that only one-half of the baryons are produced in the forward direction.

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[1] There is even some indication that in collisions where baryon excitation occurs, the fraction of energy retained by the forward excited baryon may be higher than in collisions where excitation does not take place (Kazuno, 1964). Peak and Woolcott (1965) have studied cosmic-ray interactions with 5×10^2 GeV $\leq E_p \leq 5 \times 10^4$ GeV and find indications that the nucleon retains approximately 55 percent of its initial energy; about 25 percent of the energy goes into pionization (see sec. 4–3) and the other 20 percent may go to an isobar-decay pion. With the data presented in this paper, we find that the average fraction of energy relegated to the i-process pion is 23 percent, in agreement with the conjecture of Peak and Woolcott. Thus, the Peak and Woolcott data may indicate a value of χ as high as 0.75.

We will neglect the backwardly directed baryons, since they will produce low-energy γ-rays only, and will produce them at a negligible rate compared with those produced by the more usually considered π-decay process, which we will soon discuss.

Tables 6–1 and 6–2 give the empirical data for the total cross sections of hyperons and nucleon isobars, respectively, as obtained in proton

TABLE 6–1.—*Hyperon Cross-Section Data*

Proton momentum, GeV/c	Lambda cross section, mb	Neutral sigma cross section, mb	Positive sigma cross section, mb	Reference
4.7	0.051	0.049	0.083	[a]
6	.259	.077	.....	[b]
10	.472	.196	.239	[c]
23.1	Lambda + neutral sigma cross section = 0.77			[d]
24.5	Lambda + neutral sigma cross section = 1.13			[e]

[a] Louttit et al., 1961.
[b] Chinowsky, 1966.
[c] Holmgren et al., 1967.
[d] CERN HBC and IEP groups, 1961.
[e] Bartke et al., 1963.

TABLE 6–2.—*Isobar Cross-Section Data* [a]

Incident momentum, GeV/c	Cross section, mb					Reference
	$\Delta(1.238)$	$N^*(1.410)$	$N^*(1.518)$	$N^*(1.688)$	$N^*(2.190)$	
2.85	3.8	.....	.....	.....	.....	[b]
4.55	1.5	0.63	0.68	0.7	.....	[b]
6	.376	.....	.....	.....	.....	[c]
6.06	.6	.65	.45	.5	.....	[b]
7.88	.41	.45	.31	.46	.....	[b]
10	.184	.544	.196	.562	.....	[c]
15	.142	.602	.160	.638	.....	[c]
20	.....	.660	.170	.560	0.128	[c]
30	.....	.744	.166	.576	.108	[c]

[a] For reaction channels of the type $p + p \rightarrow N^{*+} + p$ only.
[b] Blair et al., 1966.
[c] Anderson et al., 1966.

TABLE 6–3.—*Production and Decay Parameters for* Λ, Σ^+, Σ^0, N*(1.410), and N*(1.688)

[Rosenfeld et al., 1965]

Particle	Decay scheme	Mass, GeV	Production threshold energy (GeV) in p–p collision	Branching ratio for γ-producing modes	Assumed production cross section at high energies, mb	Decay time, s
$N^*(1.688)$.....	$N^{*+}(1.688) \rightarrow N+\pi^0$	1.688	1.80	[a] 0.33	0.6	$\sim 10^{-23}$
$N^*(1.410)$.....	$N^{*+}(1.410) \rightarrow N+\pi^0$	1.410	1.24	[a].33	.6	$< 10^{-23}$
Σ^+.....	$\Sigma^+ \rightarrow p+\pi^0$	1.189	1.90	.510	.8	$\sim 10^{-10}$
Σ^0.....	$\Sigma^0 \rightarrow \Lambda+\gamma$	1.192	1.90	1.00	.4	$< 10^{-14}$
Λ.....	$\Lambda \rightarrow n+\pi^0$	1.115	1.70	.331	.4	$\sim 10^{-10}$

[a] For simplicity, it is assumed that all decays occur via the i-process. More precisely, ~ 10 to 15 percent of the decays occur via the production of 2 pions.

accelerator experiments. These cross sections are related to the differential cross sections by the condition

$$\zeta_s(E_p)\sigma_s(E_p) = \int dE_s\, \sigma_s(E_s|E_p) \qquad (6\text{–}8)$$

where ζ_s is the number of particles of type s produced per collision.

Table 6–3 gives a summary of the properties of the production and decay of the hyperons and nucleon isobars with which we will concern ourselves in this calculation. In this table we have also listed the value we have assigned to the quantity $\sigma_s(E_p)$ for $E_p > 25$ GeV. We have assumed in all cases that the total cross sections have attained an asymptotic constant value above this energy. There is good evidence that this is the case for the production of the isobars that we consider (see table 6–2), but the poor data on hyperon production above 10 GeV leave some uncertainty about the validity of the assumption of constant σ_{Σ^+}, σ_{Σ^0}, and σ_Λ at high energy. Cosmic-ray studies (Wolfendale, 1963; Peters, 1962; Yekutieli, 1961; Pal and Peters, 1964; and Kazuno, 1964) indicate that the production of isobars and hyperons is significant at energies from 10^3 to 10^6 GeV. We note that $\Delta(1.238)$ and $N^*(1.518)$ production cross sections seem to decrease rapidly with increasing energy (see table 6–2),[2] whereas the isobars we have considered seem to maintain fairly constant cross sections between 5 and 30 GeV. At higher energies

[2] The cross section for $N^*(1.518)$ may reach a constant value at high energies, but it is so small compared to the others that it may be considered to be negligible.

there may be significant contributions from the production of heavier resonances (both strange and nonstrange), as well as to those in other charge states (e.g., $N^{*0}(1.688)$). However, there are indications of strong inhibition of the production of resonances involving a change in isotopic spin and involving masses greater than 2 GeV (Morrison, 1963).

In this discussion we have neglected the contribution of kaons to the cosmic γ-ray spectrum. It should not affect our results significantly for the following reasons:

(1) Kaon pairs are produced directly in the cms of the collision in the same manner as the pions of equation (6–1). Therefore, when the kaons decay through channels that produce neutral pions, they will produce the same spectral characteristics as the neutral-pion fireball component. (See sec. 6–4c.)

(2) The ratio of kaon-to-pion production is only 0.10 to 0.20 (Osborne and Wolfendale, 1963), so that only a marked difference in their production and decay schemes would affect the overall γ-ray spectrum predicted.

We note also that we have an additional contribution of Λ particles from the decay $\Sigma^0 \rightarrow \Lambda + \gamma$ to consider, apart from the Λ particles created directly. This will increase somewhat the contribution of the Λ component; however, the spectral characteristics of this component will remain unaltered.

We have not discussed the effect of the production of boson resonances such as the η, ρ, and ω mesons, as well as possible unknown resonances of higher mass. Data on such production in cosmic-ray showers are practically nil, although there is some evidence for it (Nishimura, 1967). (It is possible to speculate that the so-called fireballs, presumably created in high-energy interactions, may be either heavy boson resonances or baryon-antibaryon pairs.) (See sec. 6–4c.)

While the cross-section data for hyperon and baryon-resonance production are still sparse, we hope to have more data in the future.

6–4 THE COSMIC GAMMA-RAY SPECTRUM

A detailed discussion of the γ-ray spectra produced by the formation of various secondaries in p–p interactions has already been given in chapter 1. In general, the differential γ-ray spectrum resulting from secondary particles formed in p–p collisions is given by

$$I(E_\gamma) = \int_0^{L_{\text{eff}}} dr\, n(\mathbf{r}) \sum_s \int_{E_{\text{th},s}}^{\infty} dE_p\, I(E_p, \mathbf{r}) \int_{E_{s,\min}}^{E_{s,\max}} dE_s\, \sigma_s(E_s|E_p) \times \sum_d \zeta_d R_d f_{ds}(E_\gamma|E_s) \qquad (6\text{–}9)$$

where $\mathbf{r}$ is the radial vector from the Earth to the source, $n(\mathbf{r})$ is the density of interstellar and intergalactic gas, L_{eff} is an effective integration length, E_p is the energy of the cosmic-ray proton involved in the collision, and $I(E_p, \mathbf{r})$ is the differential flux of cosmic-ray primaries. The index s denotes a type of secondary particle produced, the index d a decay mode that leads to the production of a γ-ray, ζ_d the number of γ-rays produced in this mode, and R_d the branching ratio for decay mode d. The limits on the integrals are determined by the kinematics of the processes involved; e.g., $E_{\text{th},s}$, the lower limit on the E_p integration, is the threshold energy for the production of a particle of type s, and the limits on the E_s integration are the maximum and minimum energies that a particle of type s could have and still be able to produce a γ-ray of energy E_γ. The last quantity in equation (6–9), $f_{ds}(E_\gamma | E_s)$, is a distribution function over E_γ for γ-rays of energy E_γ produced via decay mode d by secondary particles of type s and energy E_s. Therefore,

$$\int dE_\gamma f_{ds}(E_\gamma | E_s) = 1 \tag{6–10}$$

For simplicity, we will assume that all collisions considered are those of high-energy cosmic-ray protons with hydrogen nuclei in the interstellar and intergalactic medium. The only other collisions of astrophysical significance are the p–α and α–p collisions, and we will assume here that these collisions are basically of the same character as the simpler and much more common p–p collisions, particularly in the energy region $E_p > 100$ GeV.

We will also consider here the space-averaged isotropic flux of cosmic γ-radiation, so as not to concern ourselves with the problem of variations in absolute intensity expected from various assumptions about the galactic and intergalactic cosmic-ray flux. For a discussion of these see, for example, Ginzburg and Syrovatskii (1964).

Thus, we will assume, as in chapter 5, that

$$I(E_p, \mathbf{r}) = I(E_p) \tag{6–11}$$

and

$$\int_0^{L_{\text{eff}}} dr\, n(\mathbf{r}) = \langle nL \rangle \tag{6–12}$$

In the following kinematic discussions, we will refer to the rest system of the decaying baryon by the abbreviation BARS (baryon rest system). We will also use the common relativistic convention of setting $c = 1$. Particle velocities (in units of c) are denoted by β, rest masses by M, and particle energies (in units of their respective rest masses (E/M) by γ.

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6-4a Neutral Sigma-Particle Decay

In the case of Σ^0 decay, we note that the energy of the decay γ-ray in the BARS is uniquely specified by the laws of conservation of momentum and energy. If we denote this energy by μ_Σ, it follows from equation (1–78) that

$$\mu_\Sigma = \frac{M_\Sigma^2 - M_\Lambda^2}{2M_\Sigma} = 0.0747 \text{ GeV} \tag{6–13}$$

The γ-ray energy in the ls is

$$E_\gamma = \gamma_\Sigma \mu_\Sigma (1 + \beta_\Sigma \cos \theta) \tag{6–14}$$

where θ is the angle between the γ-ray momentum vector in the BARS and the velocity vector of the BARS in the ls.

Because the decay is isotropic in θ, the resulting distribution of E_γ is uniform between the upper and lower limits allowed by equation (6–14), so that, by imposing the normalization conditions discussed previously, we find

$$f_{\gamma\Sigma}(E_\gamma | E_\Sigma) = \begin{cases} \dfrac{M_\Sigma}{2\mu_\Sigma (E_\Sigma^2 - M_\Sigma^2)^{1/2}} & \text{for } E_{\gamma 1} \leq E_\gamma \leq E_{\gamma 2} \\ 0 & \text{otherwise} \end{cases} \tag{6–15}$$

where

$$E_{\gamma 1} = \gamma_\Sigma \mu_\Sigma (1 - \beta_\Sigma) \qquad \text{and} \qquad E_{\gamma 2} = \gamma_\Sigma \mu_\Sigma (1 + \beta_\Sigma) \tag{6–16}$$

so that as $\beta_\Sigma \to 1$,

$$E_{\Sigma, \max}(E_\gamma) \to \infty$$

and

$$E_{\Sigma, \min}(E_\gamma) \to \frac{M_\Sigma}{2\mu_\Sigma} E_\gamma \tag{6–17}$$

provided the Σ^0 particles are created with an energy of the order of a gigaelectron volt or greater. We will find that this condition will be fulfilled in those cases that are of greatest interest for discussing the effects of Σ^0 decay on the cosmic γ-ray spectrum.

Incorporating equations (6–7), (6–15), and (6–17) into the general formula (6–9), as well as assumptions (6–11) and (6–12) and the branching ratio from table 6–3, we obtain

$$I_{\Sigma^0}(E_\gamma) = \frac{M_{\Sigma^0}}{4\mu_{\Sigma^0}} \langle nL \rangle \int_{E_{\text{th}\Sigma^0}}^{\infty} dE_p \, I_p(E_p) \int_{(M_{\Sigma^0}/2\mu_{\Sigma^0})E_\gamma}^{\infty} dE_{\Sigma^0} \sigma_{\Sigma^0}(E_p) \times \frac{\delta[E_{\Sigma^0} - \eta_{\Sigma^0}(E_p)]}{(E_{\Sigma^0}^2 - M_{\Sigma^0}^2)^{1/2}} \tag{6–18}$$

If we assume (as previously) a constant production cross section for $E_p > 25$ GeV and write $I_p(E_p)$ in the form

$$I_p(E_p) = K_p E_p^{-\Gamma} \tag{6-19}$$

where K_p and Γ are empirical constants, equation (6–18) can be integrated to give an approximate asymptotic spectrum for $E_\gamma > 10$ GeV. The result is

$$I_{\Sigma^0}(E_\gamma) \rightarrow K_{\Sigma^0} E_\gamma^{-\Gamma} \tag{6-20}$$

with

$$K_{\Sigma^0} \equiv \frac{K_p}{2\Gamma} \langle nL \rangle \sigma_{\Sigma^0} \left(\frac{2\chi \mu_{\Sigma^0}}{M_{\Sigma^0}} \right)^{\Gamma - 1} \tag{6-21}$$

6-4b The *i*-Process Decays

The baryons Σ^+, Λ, $N^*(1.410)$, and $N^*(1.688)$ decay by the *i*-process defined previously and thereby produce γ-rays. Because the *i*-process is a two-stage process, the determination of the distribution function $f_{di}(E_\gamma | E_i)$ requires an integration over an additional energy parameter. We will again, as in chapter 5, specify this parameter to be E'_γ. Equation (6–9) can then be expressed as (see derivation of equation (5–61)

$$I_i(E_\gamma) = C_i \int_{E\text{th},i}^{\infty} dE_p \, I(E_p) \int_{L_i}^{U_i} \frac{dE'_\gamma}{E'_\gamma} \int_{(M_i/2E'_\gamma)E_\gamma}^{\infty} dE_i \, \frac{\sigma_i(E_p)\delta[E_i - \eta_i(E_p)]}{(E_i^2 - M_i^2)^{1/2}} \tag{6-22}$$

where M_i is the mass of the isobar or hyperon considered;

$$C_i \equiv \frac{1}{2} \frac{R_i \langle nL \rangle \zeta_i}{\rho_i} \qquad \text{and} \qquad \zeta_i = 2 \tag{6-23}$$

with ρ_i defined by equation (5–55).

The relevant parameters needed to solve equation (6–22) for $N^*(1.410)$, $N^*(1.688)$, Σ^+, and Λ are given in table 6–4. If we assume, as before, that the production cross sections are constant above 25 GeV and that $I(E_p)$ can be written as $K_p E_p^{-\Gamma}$, equation (6–22) can be integrated to yield an

TABLE 6–4.—*Kinematic Parameters for* Λ, Σ^+, $N^*(1.410)$, *and* $N^*(1.688)$

Particle	μ_i, GeV	ρ_i	U_i, GeV	L_i, GeV	$C_i/\langle nL \rangle$	$(U_i/M_i)^{2.5}$
$N^*(1.688)$	0.586	0.339	0.579	0.007	0.737	6.89×10^{-2}
$N^*(1.410)$	.391	.262	.379	.012	.954	3.81×10^{-2}
Σ^+	.232	.159	.211	.022	1.60	1.51×10^{-2}
Λ	.170	.0929	.137	.033	1.78	0.58×10^{-2}

asymptotic approximation for $E_\gamma > 10$ GeV. The result is again of the form

$$I_i(E_\gamma) = K_i E_\gamma^{-\Gamma} \tag{6-24}$$

with

$$K_i \equiv \frac{2K_p C_i \sigma_i}{\Gamma^2} (2\chi)^{\Gamma-1} \frac{U_i^\Gamma - L_i^\Gamma}{M_i^\Gamma} \tag{6-25}$$

6-4c Gamma Rays From π^0-Mesons Produced by Pionization

We will now calculate the cosmic γ-ray spectrum predicted from the decay of kaons and neutral pions created by the so-called pionization or fireball processes. (See review article of Fujimoto and Hayakawa, 1967.) We have already derived the expression for the γ-ray spectrum produced from neutral pion decay (Eq. (1–100)). It then follows from assumptions (6–11) and (6–12) that

$$I_{\pi^0}(E_\gamma) = 2\langle nL\rangle \int_{E_{\mathrm{th}_{\pi^0}}}^{\infty} dE_p\, I(E_p) \int_{E_\gamma + (M_\pi^2/4E_\gamma)}^{\infty} dE_\pi \frac{\sigma(E_\pi | E_p)}{(E_\pi^2 - M_\pi^2)^{1/2}} \tag{6-26}$$

The function $\zeta_\pi(E_p)\sigma_\pi(E_p)$ is plotted in figure 5–9. In the high-energy asymptotic limit, we again assume $I(E_p) \simeq K_p E_p^{-\Gamma}$. We also assume $\sigma_\pi(E_\pi|E_p)$ to be of the approximate form

$$\sigma(E_\pi | E_p) = \sigma_0 E_p^a \delta(E_\pi - \chi_0 E_p^b) \tag{6-27}$$

with the coefficients σ_0 and χ_0 and the exponents a and b taken as constants. We then obtain from equation (6–26), for $E_\gamma > 5$ GeV, the asymptotic form

$$I_{\pi^0}(E_\gamma) = 2\langle nL\rangle \int_{E_{\mathrm{th}_{\pi^0}}}^{\infty} dE_p\, \sigma_0 E_p^a I(E_p) \int_{E_\gamma}^{\infty} dE_\pi \frac{\delta(E_\pi - \chi_0 E_p^b)}{(E_\pi^2 - M_\pi^2)^{1/2}} \tag{6-28}$$

which can be integrated to yield the solution

$$I_{\pi^0}(E_\gamma) = K_0 E_\gamma^{-g/b} \tag{6-29}$$

where

$$g \equiv (\Gamma + b) - (a + 1) \tag{6-30}$$

and

$$K_0 \equiv \frac{2}{g} \langle nL\rangle K_p \sigma_0 \chi_0^{[(g/b)-1]} \tag{6-31}$$

Pionization is a term that may be taken to include the various models that have been proposed to explain the bulk of pions produced in high-energy collisions. In these models, energy lost in the collision is con-

TABLE 6–5.—*Parameters for Determining γ-ray Spectra From Various Pionization Models* [a]

Model	Valid energy range	a	b	g/b		
				$\Gamma=2.5$	$\Gamma=2.7$	$\Gamma=3.2$
Fermi's model (Fermi, 1951).	All E_p.....	0.25	0.75	2.67	2.93	3.6
1-fireball model.....	$E_p < 150$ GeV....	.5	.5	3.0	3.4	NA
2-fireball model.....	$E_p > 10^3$ GeV....	.25	.75	NA	2.93	3.6
Landau's model.....	All E_p.....	.....	.....	(b)		

[a] See Fujimoto and Hayakawa (1967).

[b] Gamma-ray spectrum proportional to $E_\gamma^{-3.2}$, according to Lieber et al. (1965).

NA = Not applicable.

sidered to concentrate initially into one or more pion-emitting centers or fireballs. Studies indicate that the fireballs move relatively slowly in the cms compared to the baryons we have considered. Therefore, the pions arising from this process are, in general, not so energetic as *i*-process pions. Depending on the model, different multiplicity laws $\zeta(E_p)$ are predicted, all of which can be made to fit the data available in appropriate regions of E_p because of the large errors involved and the relatively slow variations of $\zeta(E_p)$ predicted over large ranges of E_p. The multiplicity laws yield the quantity a in equation (6–27). If we assume, as the data indicate, that, on the average, a constant fraction of energy goes into pionization independent of E_p, then

$$b = 1 - a \tag{6–32}$$

The exponent in equation (6–29), g/b, can then be determined for the various models, as shown in table 6–5. All the pionization spectra have steeper energy dependences (larger g/b) than the calculated *i*-process spectra (exponents equal Γ at high energy), reflecting the fact that pionization produces γ-rays of lower average energy than the *i*-process.

6–5 NUMERICAL CALCULATION OF THE ENERGY SPECTRUM OF GALACTIC GAMMA RAYS ABOVE 1 GeV

Equations (6–18), (6–22), and (6–26) were solved numerically (Stecker et al., 1968) assuming the cosmic-ray proton energy spectrum to be a broken power law of the form $K_p E_p^{-\Gamma}$, as shown in table 6–6.[3] The effect of α–p, p–α, and α–α interactions was taken into account by estimating,

[3] Table 6 in Stecker (1968a) is in error.

from the cross section and energy spectrum data given in figures 5–7 and 5–9, that p–α and α–p interactions contribute an additional flux of about 60 percent of that from p–p interactions and that α–α interactions contribute another 10 percent. Thus, we may define an effective cosmic-ray spectrum in energy per nucleon given by $K_{cr}E_{cr}^{-\Gamma}$, where $K_{cr}=1.7K_p$.

TABLE 6–6.—*Form of the Cosmic-Ray Spectrum* $I(E_{cr})=K_{cr}E_{cr}^{-\Gamma}$

Energy range, GeV	K_p	K_{cr}	Γ
10 to 10^2	$10^{-0.05}$	$10^{0.18}$	2.5
10^2 to 10^6	$10^{0.35}$	$10^{0.58}$	2.7
10^6 to $10^{8.5}$	$10^{3.35}$	$10^{3.58}$	[a] 3.2
$10^{8.5}$ to 10^{11}	$10^{-1.75}$	$10^{-1.52}$	[a] 2.6

[a] See Bradt et al., 1965.

The cross sections for the production of nucleon isobars and hyperons in p–p interactions are given in tables 6–1 and 6–2 for $E_p < 25$ GeV and were assumed to have the values in table 6–3 for $E_p > 25$ GeV.

The solution of equation (6–26) for $E_p < 150$ GeV is based on the one-fireball model. (See table 6–5.) It assumes a unique average cms pion energy independent of E_p (see ch. 5 and also Hayakawa et al., 1964) and also a multiplicity law of the form $\zeta(E_p) \sim E_p^{1/2}$. The upper limit of 150 GeV on E_p for the validity of the one-fireball model is in accordance with the treatment of Fujimoto and Hayakawa (1967). For $E_p > 500$ GeV, a two-fireball model may be taken to approximate the data more closely. As for the one-fireball model described above, equation (6–26) must be integrated numerically; the two-fireball model may be well represented by the asymptotic solution given by equation (6–29) with $a=0.25$ and $b=0.75$. (See Fujimoto and Hayakawa, 1967.) A smooth transition between the one-fireball and the two-fireball models was assumed to occur in the region $150 \leq E_p \leq 500$ GeV. It is found that the resulting solution is determined by the one-fireball model for $E_\gamma < 3.5$ GeV and by the two-fireball model for $E_\gamma > 10$ GeV.

Figure 6–1 shows the total γ-ray spectrum calculated with the use of equations (6–18), (6–22), (6–26), and (6–29), as just described as well as the results from chapter 5. Our results in figures 6–1 and 6–2 show that if we sum the contributions to the γ-ray spectrum from the pioniza-

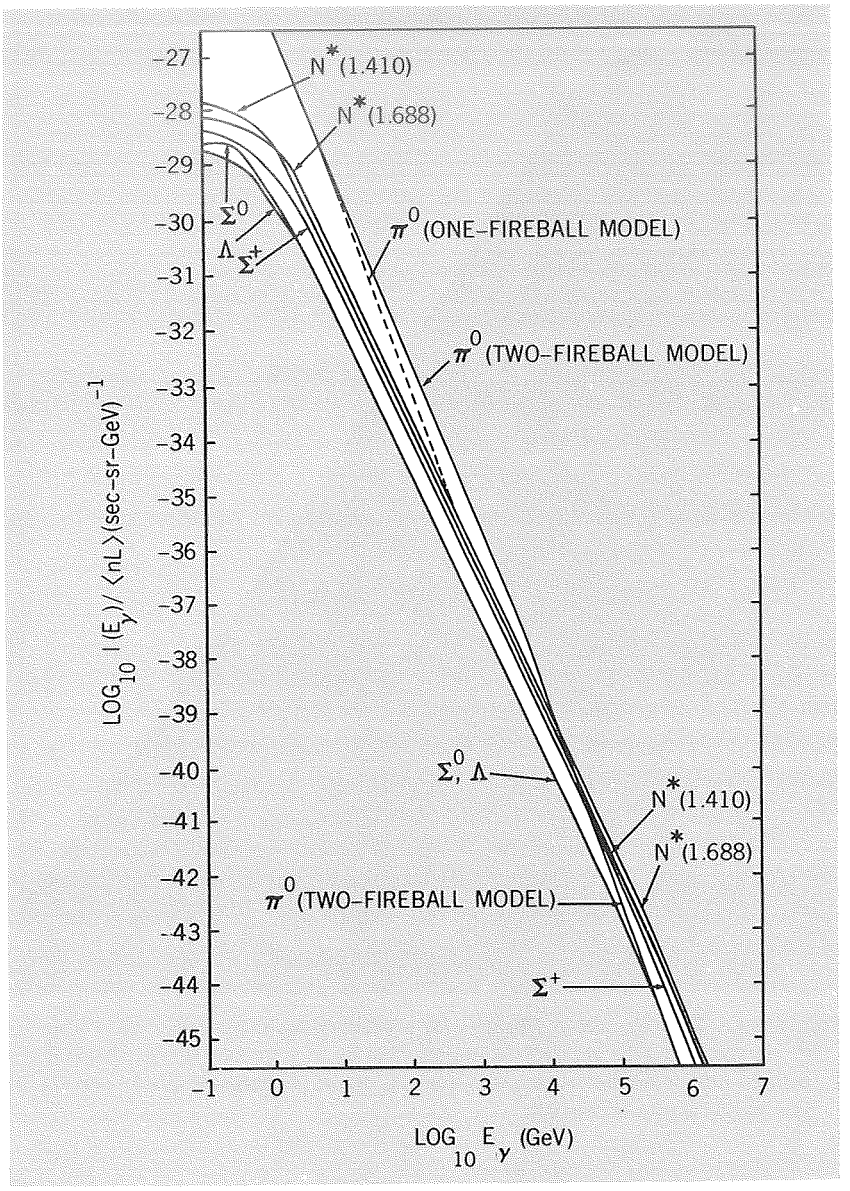

FIGURE 6–1.—Calculated γ-ray spectra from various secondary particles produced in galactic cosmic-ray interactions.

tion processes just described and from the *i*-process discussed in the previous section, we find that the characteristics of the spectrum for

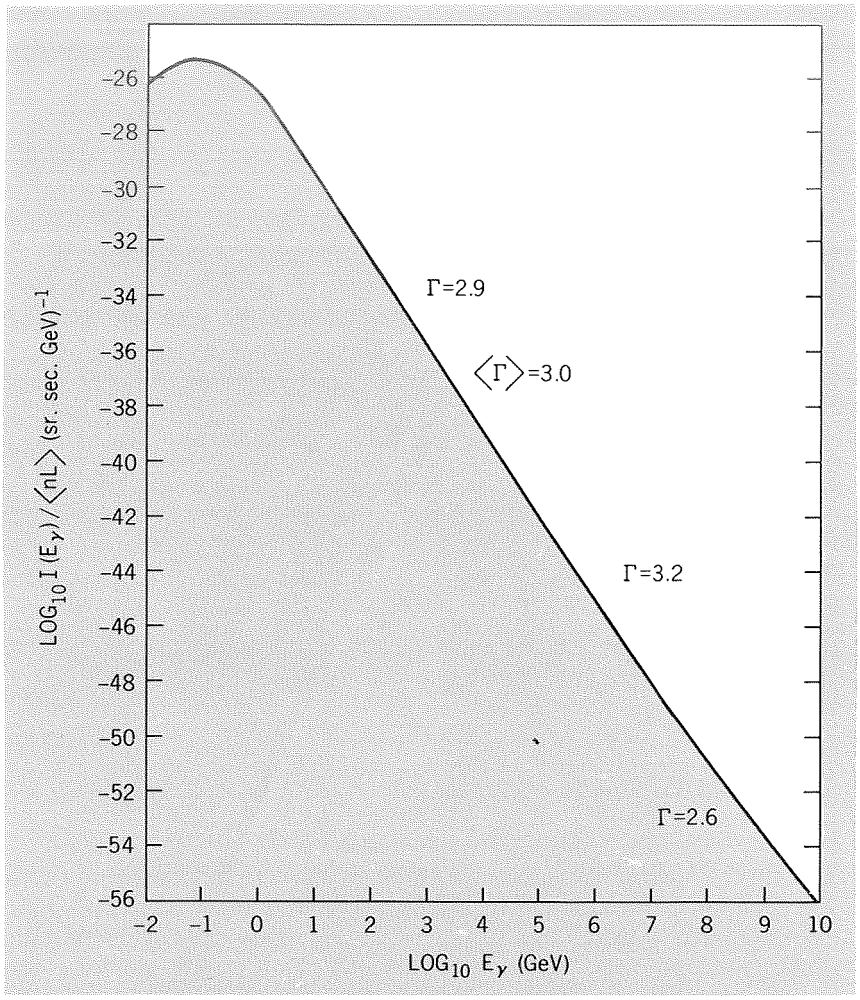

FIGURE 6–2. — Total calculated galactic γ-ray production spectrum as described in the text. The internal error in absolute intensities (due to uncertainties in experimental cross sections, cosmic-ray intensity, and branching ratios) is approximately 35 percent. The error in Γ is approximately 0.2.

$E_\gamma > 6000$ GeV may indeed be determined by the decays of various hyperons and baryon resonances.[4]

[4] For a discussion of the effect of hyperons and nucleon resonances in atmospheric cosmic-ray interactions and a comparison with the observed γ-ray and muon spectra as well as the muon charge ratio, see Pal and Peters (1964). Observational verification of the importance of the *i*-process in interactions where $E_p > 10^3$ GeV can be found in Kazuno (1964).

The characteristics of the γ-ray spectra given in figures 6–1 and 6–2 can be readily understood. The increase in the primary spectral index from 2.7 to 3.2 at 1000 TeV results in a corresponding increase in the spectral index of the *i*-process γ-ray spectra from 2.7 to 3.2 at about 200 TeV. However, the change in the primary spectrum produces a more pronounced steepening in the spectral index of the pionization γ-ray spectrum, increasing it from 2.9 to 3.6 at approximately 3 TeV. (See table 6–5.) The effect of these changes on the total γ-ray spectrum (*i*-process plus pionization) is a gradual increase in the spectral index from 2.9 to 3.2 between 3 and 300 TeV. The total calculated spectral index is approximately 2.9 between 0.1 and 1 TeV and 3.0 between 1 and 10 TeV, and is consistent with the present data on the γ-ray spectrum produced by atmospheric cosmic-ray collisions, as summarized by Fujimoto and Hayakawa (1967).